Investigating Chaos in the Nigerian Asset and Resource Management (ARM) Discovery Fund

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This paper investigates chaos in a Nigerian mutual fund, Asset and Resource Management Company Limited (ARM) for a period of eleven years. The existence of chaotic signals in the data was identified by the reconstruction of the phase space of the daily closing price of the fund and the delay time was quantified using mutual information function and the embedding dimension by the false nearest neighbours, where the values were identified to be 15 and 20 respectively. The presence of chaotic signals in the ARM data was further confirmed by the correlation dimension method which yielded a dimension of 2.2 and by the Lyapunov exponent, in which the largest Lyapunov exponent is 0.0528. The predictability of the fund was evaluated from the inverse of the largest Lyapunov exponent as 19 days.

Keywords: discovery fund, chaos, time series, lyapunov exponent, correlation dimension, correlation integral.

JEL Classification: G01, H12, P47

1.0 Introduction

Chaos can be defined as the aperiodic long-term behaviour in a deterministic system that exhibits sensitive dependence on initial conditions (Strogatz, 1994). Chaotic dynamical systems are ubiquitous in nature such as the tornado, stock market, turbulence, and weather. Their functions are different in different situations. For example, in the case of tornado, the chaotic behaviour is harmful to human beings and need to be avoided or controlled. In the case of the activities in human brain, the chaotic behaviours are useful and necessary to sustain the normal functions of brain. Thus, it is an important task to understand chaos and how it can serve human society better (Liu, 2010). The most striking feature of chaos is the unpredictability of its future.

Chaos does not mean unordered and wild. It also does not relate to randomness; instead, it is a higher-degree order. Chaos theory has been investigated in a wide range of physical experiments/researches including

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short-term movements in asset returns (Hsieh, 1993), earthquake (Sarlis and Christopoulos, 2012; Kortas, 2005), geomagnetic horizontal field (George *et al*. 2001), geomagnetic pulsation (Vorda *et al*. 1994), menstrual cycle in women (Derry and Derry, 2010), stock market (Brock et al, 1991), sunspot data (Mundt *et al.* 1991), rainfall (Sharafi *et al.* 1990), ECG and EEG (Zhang *et al.* 1992, Rapp *et al.* 1985), solar wind flow (Shollykutty Jand Kurian, 2009), traffic management (Li and Gao, 2004). The study of chaos in financial time series is significant since an underlying chaos implies long-term prediction of the system is impossible.

The concept of phase space representation rather than a time or frequency domain approach is the hallmark of nonlinear dynamical time series analysis. To recreate the phase space of the time series $x(t_0), x(t_1), ..., x(t_i), ..., x(t_n)$, we extend to a phase type of m dimensional phase space with time delay τ (Takens, 1981).

$$\begin{pmatrix} x(t_0) & x(t_1) & x(t_i) & x(t_n+(m-1)\tau) \\ x(t_0+\tau) & x(t_1+\tau) & x(t_i+\tau) & x(t_n+(m-2)\tau) \\ \vdots & \vdots & \vdots & \vdots \\ x(t_0+m(m-1)\tau & x(t_1+(m-1)\tau) & x(t_i+(m-1)\tau) & x(t_n) \end{pmatrix} \quad (1)$$

A phase point of phase space is made up of every row in equation (1). Every phase points $x(t_i)$ has m weights and embodies a certain instantaneous state and the point's trajectory of phase space is composed of the link-line of phase point. Thus the system dynamics can be studied in more dimensional phase space. There is need to estimate an optimal value of time delay and embedding dimension to obtain good representation of phase space. Time delay can be evaluated using either of autocorrelation function or mutual information (Fraser and Swinney, 1986). The average mutual information is defined as (2):

$$I(\tau) = \sum_{X(i),X(i+\tau)} P(X(i), X(i+\tau))\, log_2 \left[\frac{P(X(i), X(i+\tau))}{P(X(i))\, P(X(i+\tau))} \right] \quad (2)$$

where i is total number of samples. $P(X(i))$ and $P(X(i+\tau))$ are individual probabilities for the measurements of $X(i)$ and $X(i+\tau)$. $P(X(i), X(i+\tau))$ is the joint probability density for the measurement of $P(X(i))$ and $P(X(i+\tau))$. The appropriate time delay, τ, is defined as the first minimum of the average mutual information, $I(\tau)$. Then the values of $X(i)$ and $X(i+\tau)$ are independent enough of each other to be useful as coordinates in a time delay vector but not so independent as to have no connection with each other at all.

The sample autocorrelation of a scalar time series, $x(t)$, of N measurements is

$$\rho(T) = \frac{\sum_{n=1}^{N}(x_{n+T} - \hat{y})(y_n - \hat{y})}{\sum_{n=1}^{N}(y_n - \hat{y})^2} \tag{3}$$

where $\hat{y} = \frac{1}{N}\sum_{n=1}^{N} y_n$ is the sample mean. The smallest positive value of sampling time, T, for which the sample autocorrelation $\rho(T) \le 0$ is often used as embedding lag. Data which exhibits a strong periodic component suggests a value for which the successive co-ordinates of the embedded data will be virtually uncorrelated whilst still being temporally close.

Dimension quantifies the self-similarity of a geometrical object. There exist a few number of methods for the evaluation of the dimension of a times series box counting (Mandelbrot, 1977), Haussdorff dimension (Eckmann and Ruelle, 1985; Ott, 1993) and Grassberger and Procaccia (Grassberger and Procaccia, 1983). The algorithm proposed by Grassberger and Procaccia (1983) computes the correlation integral from which the power-law behaviour can be used to estimate the dimension of the attractor. It gives a measure of the complexity for the underlying attractor of the system. Based on this method a times series vector given by $x(t_0), x(t_1), ..., x(t_i), ..., x(t_n)$ with length N and distance less than r from each other is calculated from equation (4) according to Grassberger and Procaccia (1983)

$$C(r,m) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{i=1}^{N}\sum_{j=1}^{N} I(r - \|x_t - x_j\|) \tag{4}$$

In which the correlation integral, C(r,m), is an estimation of a probability that 2 vectors of time series with length of N, have a distance less than “r” from each other. “I” is the Heaviside function given by I(x) = 1 for x > 0 and 0 otherwise. The correlation dimension is then given by equation (5),

$$D_2 \approx \lim_{r\to 0}\lim_{N\to\infty} \frac{\log C(r,m)}{\log r} \tag{5}$$

The Lyapunov exponent is a measure of the rate of attraction to or repulsion from a fixed point in the phase space. One of the most prominent evidences of chaotic behavior of a chaotic system is the existence of positive Lyaponuv exponent. A positive Lyapunov exponent indicates divergence of trajectories in one direction, or alternaively, expansion of an initial volume in this direction, and a negative Lyapunov exponent indicates convergence of trajectories or contraction of volume along another direction. This average rate of divergence can be estimated by the method of Wolf *et al.* (1985). Another useful method commonly used is the Rosenstein *et al*. (1993) algorithm.

$$\lambda_1 = \lim_{r \to \infty} \frac{1}{t} \ln\left(\frac{\Delta x(t)}{\Delta x(0)} \right) \qquad (6)$$

Other estimators of chaoticity in a time series which we might not be able to evaluate in this research include recurrence plot, surrogate data comparison, BDS test, linear and nonlinear prediction, entropy.

It is a known fact that the investment that promotes economic growth and development requires long term funding, far longer than the duration for which most savers are willing to commit their funds. Capital market is a collection of financial institutions set up for the granting of medium and long term loans. It is a market for government securities, for corporate bonds, for the mobilization and utilization of long-term funds for development – the long term end of the financial system.

In the Nigerian capital market, participant includes Nigerian Stock Exchange, Discount Houses, Development banks, Investment banks, Building societies, Stock Broking firms, Insurance and Pension Organizations, Quoted companies, the government, individuals and the Nigerian Stock Exchange Commission (NSEC). The capital market is therefore very important to any economy because, it encourages savings and real investment in any healthy economic environment. Through the market, aggregate savings are channeled into real investment that increases the capital stock and therefore economic growth of the country.

The Nigerian Stock Exchange still has a long way to go when compared with those in some developed countries. For example, up till date it has 7 branches (Lagos, Kano, Kaduna, Ibadan, Port Harcourt, Onitsha and Abuja). As at December, 2012, there are about 198 listed with a total market capitalization of about N8.9 trillion. Many markets world over have been investigated for chaos including United States, (Hsieh, 1993); NASDAQ Composite index (Ahalpara and Parikh, 2006); Italy (Menna et al, 2002) but there has been no such research on the Nigerian stock market, probably due to paucity of data. The purpose of this research is to determine whether the time series of daily price of mutual funds in the Nigerian market is chaotic. A chaotic time series signifies that future prediction is limited on the long run.

2.0 Methodology

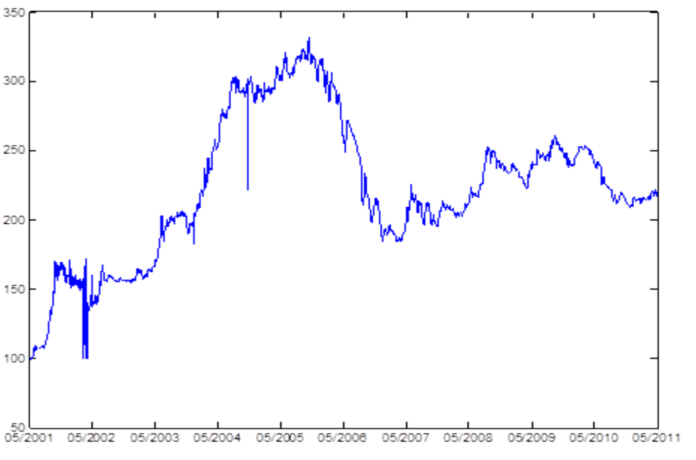

Figure 1: Time series of the ARM Discovery fund from 2001 – 2011.

The data used for this study is the daily opening price of Asset and Resource Management Company Limited (ARM) Discovery fund. The discovery fund is the flagship fund of the Asset and Resource Management Company Limited (ARM) and the oldest mutual fund in the country. The fund has returned 1027.50% on investment since inception with an average return of 24.87%. The ARM Discovery fund is a mutual fund that invests in stocks of companies quoted on the floor of the Nigerian Stock Exchange (NSE), money market instruments and real estate securities to achieve long term capital growth. Eleven (11) years data (May, 2001 – May, 2011) was obtained from the fund's website (www.arminvestmentcenter.com).

In order to reconstruct the phase space for the financial time series given by $\boldsymbol{X}_i = \boldsymbol{X}(\boldsymbol{t}_i), \quad \boldsymbol{i} = 1,\ldots, \boldsymbol{N}$ we employ the method of mutual information given by equation (2) to determine the appropriate time delay. The method of false nearest neighbour was used to obtain an estimate of the embedding dimension.

The lyapunov exponent was evaluated using the algorithm of Wolf et al (1985) given by equation (6) while the correlation integral and correlation dimension will also be evaluated. All algorithms were implemented using TISEAN (TISEAN project http://www.mpipks-dresden.mpg.de/~tisean) and Matlab.

3.0 Results and Discussion

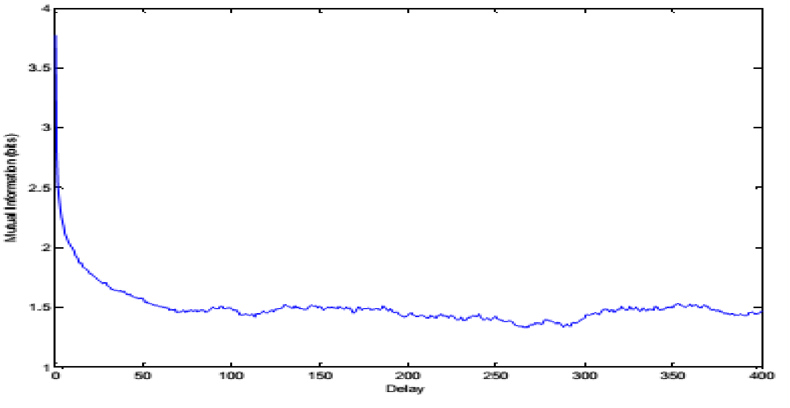

Figure 2: The mutual information of the ARM discovery fund as a function of time delay

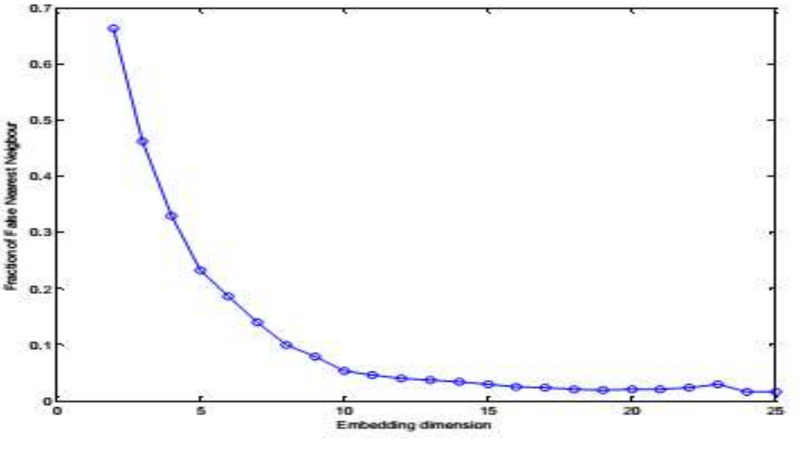

Figure 3: The fraction of false nearest neighbors as a function of the embedding dimension with $\tau = 20$.

The time series of the closing price for ARM fund is shown for the duration (May, 2001 – May, 2011) under consideration in figure (1). The closing price is seen to fluctuate with a minimum around N100 at the beginning of the study period to a maximum of N330 in May, 2005 before closing around N220 at the end of the study period. There are basically two methods (autocorrelation function and mutual information) to determine an optimal time delay.

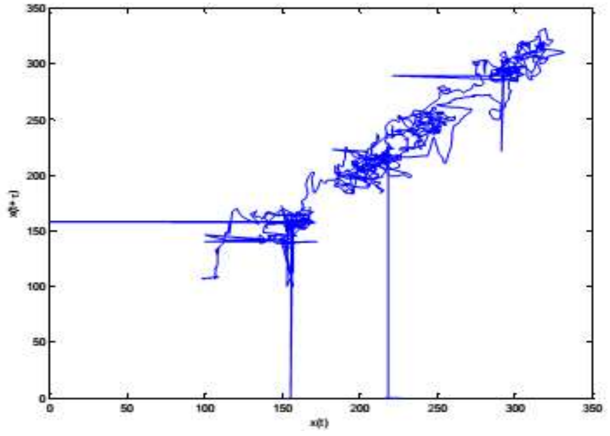

Figure 4: Phase space diagram

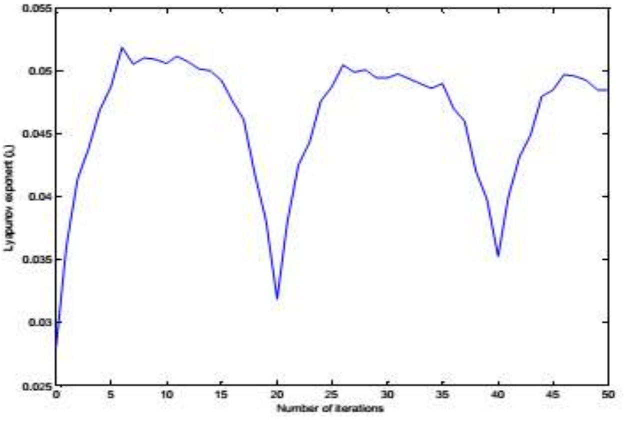

Figure 5: Lyapunov exponent of the Discovery Fund time series

The method of mutual information is used for the determination of a proper delay time (τ). If the time delayed mutual information shows a marked minimum, that value can be considered as a reasonable time delay. Hence, from Figure 2, any value greater than 15 is suitable; therefore, we choose a time delay of 20 for all analysis in this research. The method of False Nearest Neighbour is used to find an embedding dimension. The result is presented in Figure 2. Points on the graph which are have of false nearest neighbor close to zero are considered suitable as embedding lag, hence, the value of 15 was use in this work. Phase space of the fund was plotted using the value of time delay and embedding dimension obtained.

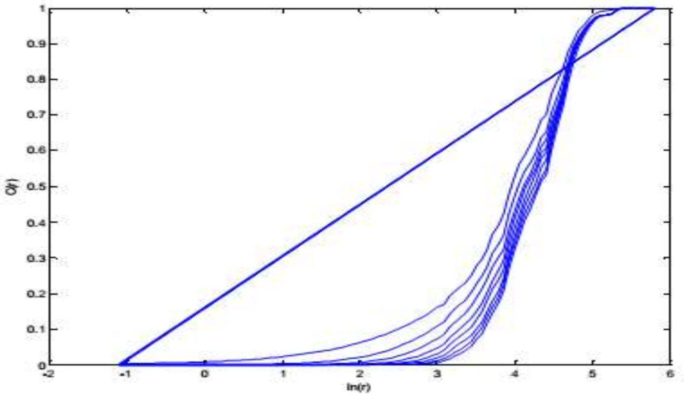

Figure 6: Correlation Integral of the ARM Discovery fund during the period under study.

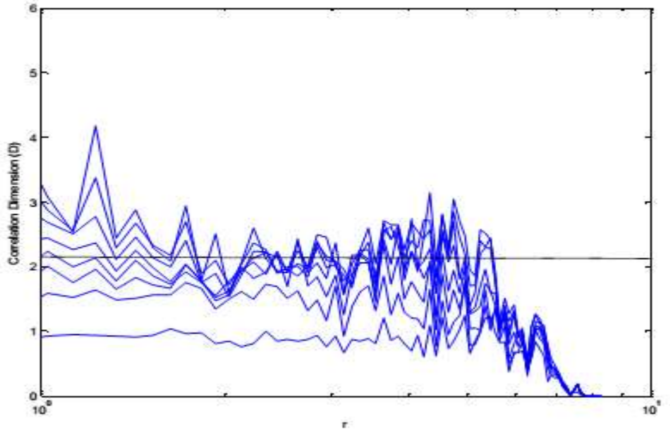

Figure 7: Correlation dimension of the ARM Discovery fund

From Figure 3, the phase space reconstruction shows convergence to a particular region which is typical of chaotic systems, in contrast to random or stochastic processes that generally cover a wide range in phase space. The phase space trajectory alone does to imply a chaotic system, hence, the need for further investigation. An indicator of nonlinearity in dynamical or time series data is the Lyapunov. A positive Lyapunov value is an indicator of chaos. From the results obtained (Figure 5) using the method Rosenstein *et al.* (1993), the Lyapunov exponent of the time series was seen to show positive exponent, a confirmation of chaos in the fund. The predictability of the fund is evaluated from the inverse of the largest Lyapunov exponent (i.e. $1/\max(\lambda)$). A consequence of the largest Lyapunov exponent is the predictability of the fund. The inverse of the largest Lyapunov exponent gives an estimate of how far into the future the prices of the fund can be predicted using linear fit. This value was calculated as 19days.

The correlation dimension of the mutual fund is shown in Figure 6. Convergence of the correlation dimension signifies a chaotic time series. The number of variables needed to describe the system effectively can be obtained from the correlation integral (Figure 7). Furthermore, the correlation integral was computed. The convergence of the correlation integral to a specific value signifies determinism in the time series. The correlation dimension was also estimated and a value of 2.2 observed.

4.0 Conclusion

In this paper, we attempt to determine whether the Discovery Fund, managed by Asset Resource and Management (ARM) is chaotic. Eleven year data (May, 2001 – May, 2011) was obtained and tested using the method of phase space reconstruction, Lyapunov Exponent estimation and Correlation Dimension.

To reconstruct the phase space, the time delay and embedding dimensions were obtained as 15 and 20 respectively, using the method of mutual information and false nearest neighbor. From the reconstructed phase space evidence of nonlinearity was seen. The presence of chaotic signals in the data was further confirmed by the correlation dimension method which yield a dimension of 2.2 and by the Lyapunov exponent which was positive, and the largest Lyapunov exponent obtained as 0.0528. The predictability of the fund is evaluated from the inverse of the largest Lyapunov exponent as 19 days.

From the results obtained, it can be inferred that ARM discovery fund is deterministic; hence, long term prediction of the fund price is impossible.

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